**CUBE HARMONIUS LABELING**

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**ABSTRACT**: In this paper we have introduced a new harmonious labelling called cube harmoniouslabelling. A graph G(V,E) with n vertices and m edges is said to be a cube harmonious graph ifthere exist an injection f: V(G)*->*{1,2,3,............$m^{3}$+1} such that the induced mapping $f^{\* }$ :E(G)$ \rightarrow ${1,8,27,...........$m^{3}$} defined by$f^{\*}$ (uv)=(f(u)+f(v)) mod ($m^{3}$+1) is a bijection.The resulting edgelabels and vertex labels are distinct. The function f is called a cube harmonious labelling of G. Here weprove that the path graph, stargraph, bistar graph and the comb graph $P\_{n} ⊙ K\_{1}$ are cube harmoniousGraphs.

KEYWORDS: Harmonious labelling, Bistar, Comb graph.

**INTRODUCTION**: In this paper, we consider finite, undirected, simple graph G (V, E) with n verticesand m edges. For notations and terminology we follow Bondy and Murthy [1]. Harmonious graphnaturally arose in the study by Graham and Sloane [3] off modular versions of additive base problems.Square graceful graphs were introduced in [4]. For a detailed survey on graph labelling we referto Gallian[2].We interested in the study of square harmonious labelling graphs by P.B.Sarasija andN.Adalin Beatress

***Definition***:

*A graph G(V,E) with n vertices and m edges is said to be a cube harmonious graph if*

*there exist an injection f: V(G)* $\rightarrow $ *{1,2,3,............*$m^{3}$*+1} such that the induced mapping*$f^{\* }$ *: E(G)* $\rightarrow $ *{1,8,27,...........*$m^{3}$*} defined by* $f^{\* }\left(uv\right)$*=( f (u) +f (v) ) mod* $ ( m^{3}+1 )$ *is a bijection.The resulting edgelabells and vertex labels are distinct. The function f is called a cube harmonious labelling of G.*

In this paper, we prove that the path graph, star graph, bistar graph and the graph

$P\_{n} ⊙ K\_{1}$are cube harmonious graphs.

**Main Results**

***Theorem****: The star graph* $K\_{1 , n}$ *is acube harmonious graph for all n ≥* 2*:*

***Proof****: Let* $K\_{1 , n}$*be a star graph with (n+1) vertices and m=n edges.*

*Let V(*$K\_{1 , n}$*)={*$v\_{1} ,v\_{2}$*,.......*$v\_{n} ,v\_{n+1}$*}.*

*Let* $v\_{n+1}$ *be the centre vertex.*

*Let E (*$K\_{1 , n}$*) = {* $v\_{i}v\_{n+1}$*,*$ 1\leq i\leq n$*}.*

*Define an injective function f:V*$\rightarrow $*{1, 2, 3,............,*$m^{3}$*+1} by*

*f(*$v\_{n+1}$*)=*$m^{3}$*+1 and*

*f(*$v\_{i}$*)=*$i^{3}$*i=1,2,........,n*

$f^{\* }$*(uv)=(f(u)+f(v)) mod (*$m^{3}$*+1).*

*f induces a bijection.*

*Hence the star graph* $K\_{1 , n}$ *is a cube harmonious graph.*

**Theorem***: The bistar graph* $B\_{p,q}$*is a cube harmonious labelling graph.*

*Proof: Let* $B\_{p , q}$ *be a bistar graph with n=p+q+2 vertices and m=p+q+1 edges.*

*Let V(*$ B\_{p , q}$*) = {* $u\_{i} , u,1\leq i \leq p, v\_{j},v,+1 \leq j\leq q$*}*

*Let B(*$ B\_{p , q}$*) = {* $u\_{i}u,1\leq i \leq p, v\_{j}v,1 \leq j\leq q$*,uv}.*

*Define an injection function f:V*$( B\_{p , q}$*) →{1,2,.......*$(p+q+1)^{3}$*} by*

*f(u)=*$m^{3}$*+1*

*f(v)=*$m^{3}$

*f(*$u\_{i}$*)=*$i^{3}$

*f(*$v\_{i}$*)=(*$m-i)^{3}$*+1,*$ 1\leq i\leq $*q.*

$f^{\* }\left(uv\right)$*= (f(u)+f(v)) mod (*$m^{3}$*+1).*

*Hence the bistar graph* $ B\_{p , q}$ *is a cube harmonious graph.*

**Theorem***: Every path* $P\_{n}$*(n≥* 3*) is a cube harmonious graph.*

*Proof: Let* $P\_{n}$*be a path with n vertices and m=(n-1) edges. LetV(*$P\_{n}$*) = {* $v\_{1} ,v\_{2}$*,.......*$v\_{n}$*} andE(*$P\_{n}$*)={*$v\_{i}v\_{i+1},1\leq i\leq n-1$*}.*

*Define an injection f: V(*$P\_{n }) \rightarrow $*{1,2,3,............*$m^{3}+1$*} by*

*f(*$v\_{1}$*)=7, f(*$v\_{2}$*)=1, f(*$v\_{3}$*)=*$m^{3}$*+1, f(*$v\_{4}$*)=*$m^{3}$*, f(*$v\_{5}$*)=*$m^{3}$*-3*$m^{2}$*+3m*

*f(*$v\_{6}$*)=f(*$v\_{5}$*) +23+6(n-6),n≥6*

*and f(*$v\_{2i}$*)=23+6*x6+6 x 8+………+6 x (2*i –* 2)+6(*i –* 2)(*n -* 2*i*)+ *f(*$v\_{2i-1}$*) , 2i ≥* 8

*f(*$v\_{7}$*)=f(*$v\_{6}$*) -66 for n=7*

*f(*$v\_{7}$*)=f(*$v\_{6}$*) - (66*+6 x 5+6 x 6+……+6 x (*n -* 3)*, n ≥* 8

*f(*$v\_{2i+1}$*)=f(*$v\_{2i}$*) - (66+6*x7+6 x 9+ …….. +6( 2i - 1)+6(i+2)+6(*i*+3)*+.......+6(n-i)),*

*2i+1 ≥* 9*.*

*To prove f is a bijection in general we consider the following cases*

$f^{\* }$*(*$v\_{4}v\_{5}$*)*=$m^{3}+m^{3}-3 m^{2}+3m-1+1 $

$$= m ^{3}+ \left( m-1 \right)^{3} +1 $$

$= (m-1)^{3} (mod)$*(*$m^{3}+1$ *)*

$f^{\* }$*(* $v\_{5}v\_{6}$*)* = $ (m-2)^{3} (mod)( m^{3}+1)$

for n=7

$f^{\* }$*(* $v\_{6}v\_{7}$*)* =$ (m-3)^{3}\left(mod\right) (m^{3}+1$)

for n≥ 8

$f^{\* }$*(* $v\_{6}v\_{7}$*) ≡* $ (m-3)^{3}\left(mod\right) (m^{3}+1$)

for n=8 and n≥8

$f^{\* }$*(* $v\_{7}v\_{8}$*) ≡* $ (m-4)^{3}\left(mod\right) (m^{3}+1$)

$f^{\* }$*(* $v\_{2i }v\_{2i+1}$*) ≡* $ (m-2i-3)^{3}\left(mod\right) (m^{3}+1$) for 2i *≥ 8, 2i+1≥9*

*Hence f induces a bijection* $f^{\* }:E ( P\_{n } ) \rightarrow \{ 1,8,27,……………., m^{3}\}$

$f^{\* }\left(uv\right)=\left( f\left(u\right)+f\left(v\right)\right)mod ((m^{3}+1)$*.*

*The edge labels are distinct.*

*Hence every path* $P\_{n, }, n \geq 3$*is a cube harmonious graph.*

**Theorem***: The comb graph* $P\_{n} ⊙ K\_{1}\left(n \geq 2 \right) i$*s a cube harmonius graph.*

***Proof****: Let{*$u\_{1} ,u\_{2},$ *……..*$u\_{n}$ *} be the vertices of the path* $P\_{n, }$*and{* $v\_{1} ,v\_{2}$*,.......*$v\_{n} \}$ *be the pendent vertices at*$u\_{1} ,u\_{2},$*………* $u\_{n}$*respectively.*

*Here m=2n - 1.*

*N is the total number of vertices of* $ P\_{n} ⊙ K\_{1}$

*Define an injection f : V (*$P\_{n} ⊙ K\_{1}$*) → {1,2,3,.......*$ (2n-1)^{3}+1$ *by*

*f(*$u\_{1}$*)=7,f(*$u\_{2}$*)=1, f(*$u\_{3}$*)=* $m^{3}+1$

*f((*$u\_{4}$*)=*$m^{3}$*f(*$u\_{5}$*)=* $m^{3}-3 m^{3}+3m $

*f(*$u\_{6}$*)=f(*$u\_{5}$*) +23+6(N – 6), n*$ \geq $*6*

*and f(*$u\_{2i}$*)=23+6*x6+6 x 8+……..+6 x(2i – 2)+6(i - 2)(*N –* 2i)+ *f(*$u\_{2i-1}$*) , 2i* $\geq $8

*f(*$u\_{7}$*)=f(*$u\_{6}$*) -66 for N=7*

*f(*$u\_{7}$*)=f(*$u\_{6}$*) - (66*+6 x 5+6 x 6+……….+6 x (*N -* 3)*), n*$ \geq $8

*f(*$u\_{2i+1}$*) =f(*$ u\_{2i}$*) - (66+6*x7+6 x 9+ …+6(2*i -* 1)+6(*i*+2)+6(*i*+3)*+......+6(N-i)),*

*for 2i+1*$ \geq $9*.*

*f(*$v\_{1}$*)=*$(n+2)^{3}-7$

*f(*$v\_{2}$*)=* $(n+1)^{3}-1$

*f(*$v\_{3}$*)=* $(n)^{3}$

*f(*$v\_{4}$*)=* $(n-1)^{3}+1$

*f(*$v\_{i}$*)=* $\left(m^{3}+1\right)-f\left(u\_{ i}\right)+(n-(i-3))^{3}$ *for i*$\geq 5$

*The induced function* $f^{\* }:$ *:E(*$P\_{n} ⊙ K\_{1}$*) → {1, 8, 27,...........*$m^{3}$*} is bijective.*

*Hence the comb graph is cube harmonious graph.*

REFERENCES

[1]. J.A.Bondy and U.S.R.Murthy, Graph theory with Applications, Macmillian, London,1976.

[2]. J.A.Gallian, A dynamic survey of graph labelling, the electronics J. of Combinatorics, 16, 2009.

[3]. R.L.Graham and N.J.A.sloane, on additive bases and harmonious graphs, SIAM J.Alg.Discretemath., 1(1980)382-404.

[4]. P.B.Sarasija and R.Binthiya, Even harmonious graphs with applications, International JournalOf Computer Science and Information security. Vol.9, No.7, (2011)161-163.

[5]. T.Tharmaraj and P.B.Sarasija, Square graceful graphs, international journal of MathematicsAnd Soft Computing Vol.4 No.1.[2014], 129-137

[6]. T.Tharmaraj and P.B.Sarasija, Some Square graceful graphs, international journal of MathematicsAnd Soft Computing Vol.5 No.1.[2014], 119-127

[7]. P.B.Sarasija and N.Adalin Beatress, Square harmonious graphs, International Journal of

Advanced Research in Science, Engineering and Technology Vol.3, issue 2, February (2016).

[8]. P.B.Sarasija and N.Adalin Beatress, Even - Odd harmonious graphs, international journal ofMathematics and Soft Computing Vol.5 No.1.[2015], 23-29.